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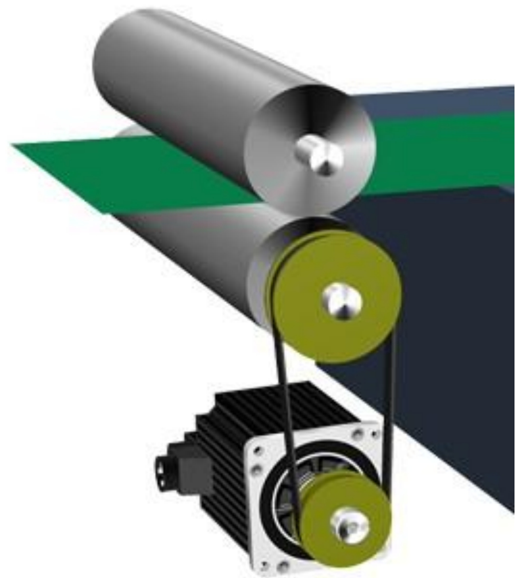
Mechanical design for servos places an additional set of constraints to the design rules normally used for power transmission design. These added constraints relate primarily to the stiffness of the system and inertial matching. Decisions on speed reducers, couplings, shaft configurations and many other critical aspects of the mechanical design are often made very early in the design process. Once made, these decisions can be very expensive and time consuming to change.

The intent of this technical note is to communicate some design information and "rules of thumb" that we at ORMEC have found important in our many years of applying servos to industrial automation. Are these guidelines universal truths? Obviously any set of design rules will from time to time collide with a special case. However, the following guidelines will apply in the vast majority of cases and the prudent designer will only violate them after careful analysis and with a thorough understanding of the risks involved.

Why be concerned about the load?

Knowledgeable servo designers are wary of using a servomotor to drive mechanisms whose moment of inertia is many times that of the motor itself. However, economic pressures and other technical advantages often cause engineers to want to direct drive high inertia loads. The main advantages they seek are to eliminate the cost, the maintenance and the inaccuracy of a reducer. While it is usually easier to avoid large inertia mismatches, with appropriate attention to detail, they can be made to work.

No designer should approach the design of a servo driven mechanism especially one with a significant inertial mismatch, without doing a careful analysis of the natural frequency of the mechanical system. "Seat of the pants" engineering is almost guaranteed to result in problems.



One reason many designers lean towards direct drive, is to avoid the cyclical inaccuracies that gear reducers can introduce. The closed loop servo can monitor its actual speed and position and rapidly adjust for load disturbances. When the load inertia is many times the motor inertia, the motor has only a very small amount of kinetic energy compared to the load. To compensate for a sudden change in load, the servo amplifier must inject a large amount of energy into the servomotor very quickly. This demands a high gain, high bandwidth system. When you combine high gain, high bandwidth and large inertia mismatches, alarm bells should start to sound.

What constitutes a large inertia mismatch? At one time designers strove to achieve a 1:1 inertia match. They considered anything above 5:1 to be a potential problem. The application of digital technology to servo control, digital signal processors in particular, has relaxed that constraint. These days, mismatches of 100:1 or even 1000:1 can be made to work with careful mechanical design. While good mechanical design is always important, inertia mismatches of 10:1 and above, can only work if the mechanical designer has paid careful attention to minimizing backlash and compliance in the design.

Backlash Effects

Backlash, sometimes called "lost motion", is a mechanical effect that allows you to turn the motor shaft without causing any motion at the load. Generally, with the servo drive disabled, you can rotate the motor shaft back and forth over a limited range. If you release the shaft, it will stay where it is. Backlash has the effect of temporarily uncoupling and recoupling the load and the motor with changing speed and direction. When stopped, if there are insufficient external forces acting on the load, the motor may for all intents and purposes be disconnected from the load completely. If the servo is tuned to work well when the load is disconnected, it will have extremely poor performance when the load is connected. Likewise, if the servo is tuned for adequate performance, it will be unstable when the backlash disconnects the load.

The most common symptom of backlash is a "buzz" , often very loud, which occurs primarily when the motor is stopped. Often you can eliminate the "buzz", the system becomes very "soft". Sometimes the gain is so low that it cannot stabilize the position loop and the system may oscillate wildly at a low frequency (1-5Hz). The only way to solve the problem is to mechanically eliminate the backlash. If you do not eliminate the buzz, over time it may overheat the motor or ruin the mechanical system. A common source of backlash is using a keyway or set screw to couple to the motor shaft. While keyways are fine for lawnmowers, they are inadequate for high performance servos! A clamp style coupling, preferably a taper lock bushing, is the only acceptable way to couple to a servomotor shaft.

Another common cause of backlash is improperly adjusted spur gears or using gear reducers that are not designed for servo applications. Properly selected precision planetary gearheads, such as ORMEC GBX Series are generally quite good for servo applications. However, you must make sure the gearhead and primary pinion are mounted and adjusted properly. If you are not careful, improper mounting will introduce backlash into the system and it will deteriorate over time.

Compliance Effects

Compliance also allows you to rotate the input shaft without the load moving. However, with compliance you are actually "winding up" the mechanical system like a spring. When you release the input shaft, it will spring back close to its original position.

Compliance, or wind up, effects show up as a torsional resonant frequency which in turn causes the servo to be unstable. The instability generally shows up as a medium to high frequency oscillation in the order of 100 to 500 Hz. Unlike the "buzz" caused by backlash the sound is often a pure note and does not go away when the motor moves. The frequency does not change as you manipulate the servo tuning however the amplitude may change. Applying a friction load may also reduce the amplitude of the oscillation. As with the buzz caused by backlash, left uncorrected, this resonance will overheat the motor and possibly damage the mechanism.

Long drive shafts, where the bulk of the load inertia is some distance from the motor, are a common cause of this type of problem. It is often surprising how much windup can exist in what appears to be a rather substantial shaft. Take for example a one inch diameter stainless steel shaft about 18 inches long. When you apply a 500 in-lb load the shaft will wind up almost 0.5 degrees (see equations in Figure A above).

If we take that same shaft and connect an HB200 motor, with a moment of inertia of 0.0407 in-lb-s^2, on one end and a load inertia of 100 times the motor inertia on the other end, the natural frequency of the system will be about 184 Hz (see equations in Figure A above).

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Generally, if the natural frequency of the system is less than about 500 Hz, you may encounter resonance problems in high performance systems. There is no magic about the number 500, it is simply a rule of thumb. With a mechanical natural frequency above 500 Hz, you are unlikely to have a resonance problem. At frequencies below 500 Hz, the probability of resonance problems increases. In the above example, to achieve a natural frequency of 500 Hz, you would have to increase the shaft diameter to 1.6875 inches or decrease

EQUATIONS FOR SOLID SHAFT RESONANT FREQUENCY

Figure A) **Equations for shaft windup**

Stiffness $S = \dfrac{\pi(OD^4 - ID^4)G}{32L}$ in-lb/rad

Windup $W = \dfrac{360T}{2\pi S}$ degrees

Where:		
	OD	is shaft outside diameter (inches)
	ID	is shaft inside diameter (inches)
	L	is shaft length (inches)
	T	is applied torque (in-lb)
	G	is shaft shear modulus (lb/in^2) [11 x 10^6 for stainless steel]

Figure B) **Equations for natural frequency**

Natural Frequency $f = \dfrac{1}{2\pi}\sqrt{S\,\dfrac{(J_M + J_L)}{(J_M * J_L)}}$

Where:		
	J_M	is motor inertia (in-lb-s^2)
	J_L	is load inertia (in-lb-s^2)
	S	is shaft stiffness (in-lb/rad) [see equations above]

Figure C) **Equation for accumulated stiffness 'S'**

$$S_T = \frac{1}{\frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} + \cdots \frac{1}{S_n}}$$

the shaft length to 2.5 inches. The natural frequency will usually be determined by the least stiff portion of the drive train which is often the shaft coupling. Be sure to obtain stiffness specifications for any coupling you expect to use and complete the necessary calculations. If you have more than one "unstiff" component in your drive train, the effects are additive in that the resulting overall stiffness is given by equation in Figure C in the chart on page 2.

If the moment of inertia of components located between these couplings is significant compared to the overall load inertia, the calculations become a lot more complex and usually result in multiple resonant frequencies. Often times, choice of couplings, shaft dimensions and attachment methods have surprising effects. No designer should approach the design of a servo driven mechanism, especially one with a significant inertial mismatch, without doing a careful analysis of the natural frequency of the mechanical system."Seat of the pants" engineering is almost guaranteed to result in problems.

Coupling Selection

In any servo mechanism, selection of mechanical couplings is critical. When there is a large inertia mismatch it is doubly so. Many times it is the choice of coupling that causes the system to have a low resonant frequency. Helical style couplings are almost never stiff enough to avoid problems unless the load inertia is so low as to be insignificant. The best choice is a bellows style coupling with taper lock bushings.

If we take a typical inexpensive helical coupling rated for 500 in-lbs of torque, the stiffness will be approximately 72×10^3 in-lb/rad. If we use this coupling on the load system described earlier, it will limit the system natural frequency to less than 197 Hz. Clearly this type of coupling would not be adequate. So instead, if we take a similarly rated bellows coupling1 , its stiffness will be 433×10^3 in-lb/rad. This coupling would have a natural frequency of 480 Hz, which is much less likely to affect operation.

Generally, it is best to avoid helical, disc, oldham, split beam and jaw type couplers. Metal bellows will usually provide the best results. In addition to the coupling type you must also pay careful attention to how it is attached to the shafts. A clamping or taper lock is the best way to go. Always avoid keyways and set screws.

How to Tame Mechanical Load Problems

As said earlier, most mechanical load problems are really backlash and/or compliance problems. The solutions involve changing the mechanical design to eliminate any backlash and to raise the natural frequency above 500 Hz. Increasing the natural frequency can often be accomplished by selecting a stiffer coupling, increasing the diameter of shafts or decreasing the lengths of shafts.

Another way to reduce the possibility of instability is to add a speed reducer. This step can reduce the reflected inertia by the square of the reduction ratio. Adding speed reduction also increases resolution at the motor and improves performance at low load speeds. It also allows the motor to run at a higher speed which gives it more kinetic energy to overcome load disturbances. This in turn can reduce the gain and bandwidth requirements for the servo. Obviously, adding a speed reducer adds the reducer's efficiency losses, inaccuracies and compliance to the system so careful selection of the reducer type is critical.

Another helpful technique, although one with its own disadvantages, is to add notch filtering in the servodrive command. Ideally, a notch filter exactly counters the effect of the mechanical resonance and eliminates the system's ability to respond at that frequency. When properly designed and implemented, they work well without requiring mechanical changes. The disadvantages of notch filters are:

- The filter will only work if the mechanism does not undergo significant changes over time. As mechanisms wear or heat up, their natural frequencies can change. The natural frequency will also change as the load inertia changes. A once stable system may become unstable if the natural frequency shifts enough that the notch filter no longer cancels it out.
- Many resonant loads have several natural frequencies. If you design a notch filter with a wide enough notch to cover all of the natural frequencies, you may end up with what is effectively a low pass filter which will reduce servo response considerably.

While many mechanical problems can be resolved using notch filters, they don't address the root cause of the problem and therefore are not a universal cure all.

Timing Belts

Timing belts are a very economical and surprisingly accurate way to provide modest speed reductions. For servo applications, you should choose a belt with a high tensile stiffness and low backlash. Belts that use an aramid tensile member and a modified curvilinear tooth profile are good in both qualities. Belt selection and design is a fairly specialized process and the reader would be well advised to consult one of the many excellent application guides published by belt manufacturers for assistance in this area. Another advantage of timing belts over other types of reducer is their very high efficiency, 95% or better. A disadvantage of timing belts is the added inertia of the pulleys. However, the added inertia can be minimized by modifying standard pulleys to reduce their mass. Custom pulleys made from light weight materials such as aluminum are available from most belt manufacturers.

Timing belts designed for precise positioning have a tensile member that uses fibers with a very high tensile strength.

TIMING BELT EQUATIONS TO DETERMINE FREQUENCY

Figure D)

$$\text{Mass} \quad M_1 = \frac{J_1}{(R_1)^2} \quad \text{lb}$$

$$\text{Mass} \quad M_2 = \frac{J_2}{(R_2)^2} \quad \text{lb}$$

$$\text{Belt Stiffness} \quad S = \frac{EA * \text{width}}{\text{span}} \quad \text{lb/inch}$$

$$\text{Natural Frequency} \quad F = \frac{1}{2\pi}\sqrt{S\,\frac{(M_1 + M_2)}{(M_1 * M_2)}}$$

Where:

- J_1 is the total moment of inertia at the driving pulley (in-lb-s²). It includes the inertia of the pulley and everything connected to it.
- J_2 is the total moment of inertia at the load pulley (in-lb-s²). It includes the inertia of the pulley and everything connected to it.
- R_1 is the radius of the driving pulley (inches).
- R_2 is the radius of the driven pulley (inches).
- S is the belt stiffness (lb/inch).
- span is the belt span, which is the distance the belt spans between the initial contact points on the pulleys on the tension side of the belt (inches).
- width is the belt width (inches).
- EA Belt spring rate in (lbs/inch width per unit strain).

These fibers are set at a diameter that is much larger than a typical direct drive shaft. If the belt system has been properly sized the stiffness of the system can be better than a solid steel shaft. In the example shown on page 2, the system had a shaft windup of 0.5 degrees with a 500 in-lb load. If you substitute a 37mm wide timing belt[2] using 6 inch pulleys[3] on 18 inch centers, the windup will be less than 0.25 degrees. When a speed reduction is used rather than 1:1, the windup decreases further. To calculate the natural frequency of a timing belt system, you need to know the spring rate of the belt. This is available from the belt manufacturer and is normally called the EA factor. The EA factor for a belt varies with the tension of the belt and is usually shown on a chart that plots EA (lbs per inch width per unit strain) against belt load (lbs per inch width). The calculations for windup and resonant frequency of a timing belt system can get quite tricky since you must take belt tension and load forces into account when deciding what EA value to use[4]. Unless you are already familiar with the techniques, you should seek the assistance of your belt supplier. For the purpose of this application note it is sufficient to say that it is not difficult to design a timing belt drive that is as stiff or stiffer than a typical direct coupled load. The main advantage of a timing belt system is not that it significantly increases the natural frequency but rather it changes the amplitude of the resonance. A timing belt system adds considerable damping to the system and for a given natural frequency, will allow higher gain settings before resonance becomes a problem. Another advantage is that it allows you to run the motor at a higher speed which, if the motor inertia is small compared to the load, will provide better operation.

One thing to remember is that resonances are usually a greater problem when the system is stopped than when it is moving under load. By reducing belt tension slightly, you can provide a measure of decoupling between the motor and load. This decoupling and the damping provided by the belt will often reduce resonance problems. Be careful not to reduce the tension too much or accuracy will suffer. A good rule of thumb is to make sure the “slack” side of the belt is always under some tension. Another thing to remember is that too much belt tension can easily generate a radial load on the motor shaft which will drastically reduce bearing life. When belt tension must be high, always use a jack shaft with its own bearings to isolate the motor shaft from the radial load.

Summary

If you have a load to motor inertia mismatch greater than 10:1, or have a significant portion of the load inertia coupled through long shafts, you will need to carefully analyze your mechanical design. You will need to make sure there is no backlash and that the natural frequency is higher than 500 Hz. If you cannot achieve that, gear or belt reduction are the best alternatives for making it work. As a last resort, notch filtering may be practical in some special cases.

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2. Gates Rubber Co. Poly Chain GT [Part # 14M-1400-37]
3. Gates Rubber Co. [Part # 14M-34S-37]
4. Selecting Synchronous Belts for Precise Positioning, by A.W. Wallin - Applications Engineer, Synchronous Drives Div., The Gates Rubber Co., Denver CO.

For more information

Contact the motion control experts at ORMEC for further information. Please call us at (585) 385-3520 or email sales@ormec.com Visit our website at www.ormec.com